A SIMPLE PROBABILISTIC MODEL OF BUBBLE COLUMN

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A probabilistic model of bubble column was derived from a deterministic model so that non--correlated or autocorrelated noise was added at the column outlet. A pseudo-random binary series of maximum length – PRBS was used as input signal. The effect of noise was observed in terms of impulse characteristics calculated by the correlation method from the responses of model to the PRBS input. The possibilities of PRBS when simulating the behaviour of bubble column were judged in the work, and the effect of magnitude of noise amplitude on the form of impulse characteristics was evaluated.

As a mathematical model we consider a set of different types of algebraic and differential equations which can be formulated as deterministic or probabilistic. The deterministic models are those, independent variables, dependent variables and parameters of which a certain number or series of firmly given numbers is assigned to. On the contrary, the variables and parameters of the probabilistic models are random quantities. The probabilistic models are more complex but in good many cases accord better with the substance of modelled phenomena because a random component can appear in the course of process in the form of non-measurable conditions of surroundings or random fluctuation of system parameters. The deterministic models cope with this fact by adding safety factors to the calculated system parameters so that the range of parameters is covered but the added values are often unnecessarily high. With the probabilistic models we try to obtain the corresponding statistical description of output variables for the given random inputs and parameters. The probabilistic models were developed in two different ways. In the first way, the given process is divided into smaller subsystems, and probabilistic relations are employed for their description. In such a case, a certain model of system is assumed. The statistical properties of its smaller parts are either assumed or found out by means of experimental methods. The mathematical description of the model containing the probabilistic relations employs then statistical properties of these smaller parts to obtain the description of some other variables of the model. The second way stems from the deterministic model. The probabilistic model is formed from it by introducing the random variables in the place of inputs, parameters, boundary and initial conditions. This work investigates, by means of a model of bubble column, the effect of non-correlated and autocorrelated noise on its behaviour.

THEORETICAL

The bubble column with a stagnant liquid layer and flowing gas phase was described by the axial dispersion model taken from the work by Mangartz and Pilhofer[1]. The gas phase is described by Eq. (*1*),

$$\frac{\partial c_g}{\partial t} = D_g \frac{\partial^2 c_g}{\partial z^2} - U \frac{\partial c_g}{\partial z} - \frac{k_l a_l (1 - \varepsilon_g)}{\varepsilon_g} (\chi c_g - c_l)\,, \qquad (1)$$

and the liquid phase by Eq. (*2*),

$$\frac{\partial c_l}{\partial t} = k_l a_l \left(\chi \frac{1}{L} \int_0^L c_g \, dz - c_l \right). \qquad (2)$$

The boundary conditions are expressed by Eqs (*3*) and (*4*), the initial conditions by Eqs (*5*) and (*6*).

$$z = 0: U c_i = U c_g - D_g \frac{\partial c_g}{\partial z} \qquad (3)$$

$$z = L: \frac{\partial c_g}{\partial z} = 0 \qquad (4)$$

$$t = 0: c_g = c_{g1} \qquad (5)$$

$$c_l = c_{l1} \qquad (6)$$

Equations (*1*) and (*2*) with boundary and initial conditions (*3*)–(*6*) represent the axial dispersion model with mass transfer. This model was verified experimentally[2] in terms of PRBS method. In this work we aimed at the study of the gas phase and therefore the input signal was realised by means of concentration c_i in boundary condition (*3*). In addition to model with mass transfer we considered also the model without mass transfer. The gas phase is in this case described by Eq. (*7*),

$$\frac{\partial c_g}{\partial t} = D_g \frac{\partial^2 c_g}{\partial z^2} - U \frac{\partial c_g}{\partial z}\,. \qquad (7)$$

It can be seen from its form that the gas phase can be modelled separately from the liquid phase. The boundary conditions are given by Eqs (*3*) and (*4*), the initial condition by Eq. (*5*). For the calculations, the equations were used in a dimensionless form. The conversion to the dimensionless form was carried out on the basis of Eqs (8)–(11).

$$c_g^* = (c_g - c_{g1})/(c_{g2} - c_{g1}) \quad (8)$$

$$c_l^* = (c_l - c_{l1})/(c_{l2} - c_{l1}) \quad (9)$$

$$t^* = tU/L \quad (10)$$

$$z^* = z/L \quad (11)$$

The pseudo-random binary series of maximum length – PRBS was used as the input signal of both models. It is a determined periodical signal acquiring only two values symmetrically placed around zero. The detailed description of PRBS and its properties can be found in the literature[2–9]. The PRBS was used here for determining the model impulse characteristic. The impulse characteristic was chosen for the fact that, according to its form, it is possible to identify unambigously the model with mass transfer and/or the model without mass transfer. Besides, the calculation of impulse characteristic from the responses of model to PRBS meets our purpose because the output signal can be modified, before calculating the values of impulse characteristic, by adding noise so that the resulting curve already includes the effect of noise. The calculation represented first the change of c_l in boundary condition (3) according to the prescription of the PRBS chosen. By numerical solving the model equations, the response of models to this input signal was then calculated. The impulse characteristics were calculated from the responses obtained by the correlation method[3]. The relations for calculating the ordinates of impulse characteristic $h(0)$ and $h(i\,\Delta t)$ are Eqs (12) and (13).

$$h(0) = \frac{2}{\Delta t \bar{a}^2} \frac{N}{N+1} \left[R_{xy}(0) + \sum_{j=0}^{N-1} R_{xy}(j\,\Delta t)\right] \quad (12)$$

$$h(i\,\Delta t) = \frac{1}{\Delta t \bar{a}^2} \frac{N}{N+1} \left[R_{xy}(i\,\Delta t) + \sum_{j=0}^{N-1} R_{xy}(j\,\Delta t)\right], \quad i = 1, 2, \ldots N-1 \quad (13)$$

The pseudocorrelation function $R_{xy}(k\,\Delta t)$ was calculated by means of the series (14),

$$R_{xy}(k\,\Delta t) = \frac{1}{N} \sum_{i=0}^{N-1} x[(i-k)\,\Delta t]\, y(i\,\Delta t)\,. \quad (14)$$

In the presence of noise at the column outlet, the impulse characteristics were calculated from the input signal of ideal PRBS and the sum of response of the model to this signal and amplitude of noise as it is schematically illustrated in Fig. 1. The noise amplitude was expressed relatively to the amplitude of the signal used. Since the input signal of model was PRBS, we defined quotient q of the noise amplitude and the distance of desired signal from the centre to the top as it can be seen in Fig. 2. The added noise was non-correlated one with normal distribution and autocorrelated noise. The non-correlated noise generator (taken from the scientific program library of IBM 360 computer[10]) generates normally distributed random numbers for the given mean value and standard deviation. The program for generating the autocorrelated noise was made up according to the directions in the work by Berryman[11].

DISCUSSION

The first part of results was related to the axial dispersion model with mass transfer. The noise amplitude was chosen up to 0·1 of magnitude of PRBS ordinate so that quotient q was in the range of 0–0·1. The mean value of non-correlated and autocorrelated noise was always 0 and the standard deviation 1. When choosing the PRBS period, we took into account all limitations and mutual feedback N (period of PRBS) and Δt (basic time interval)[3]. We chose PRBS with $N = 31$ and $\Delta t = 16$ s which was used in the experimental verification of the chosen model of bubble column[2] and PRBS with nearest higher value of period ($N = 63$). The value of $\Delta t = 8$ s was attached to this value of N. Quotient q was increased from 0 always by 0·01 up to the value of 0·1. For each PRBS, we calculated, by using the correlation method, also the impulse characteristic of model without adding noise to be able to compare with it the impulse characteristics with non-zero value of q. All the coefficients in model equations were evaluated from the values of parameters inserted in agreement with the conditions of experiments carried out to verify the model chosen[2]. The impulse characteristics in Fig. 3 were calculated from the input signal of PRBS at $N = 31$, $\Delta t = 16$ s and the sum of model response and non-correlated noise with $q = 0$, 0·01, 0·02, 0·03, 0·04, 0·05 (Fig. 3a) and with $q = 0$, 0·06, 0·07, 0·08, 0·09, 0·1 (Fig. 3b). The impulse characteristics in Fig. 4 stem from the same parameters of PRBS, the added noise was autocorrelated with $q = 0$, 0·01,

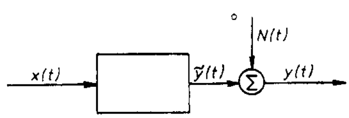

FIG. 1

Schematic illustration of probabilistic model. $x(t)$ is the input signal, $\tilde{y}(t)$ is the output deterministic signal, $y(t)$ is the output signal, $N(t)$ is the random signal (noise)

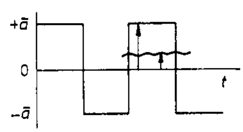

FIG. 2

Schematic illustration of quotient q of noise amplitude and distance of desired signal from the centre to the top. $+\bar{a}$ is the upper level of PRBS, $-\bar{a}$ is the lower level of PRBS, t is time

0·02, 0·03, 0·04, 0·05 (Fig. 4*a*) and with q = 0, 0·06, 0·07, 0·08, 0·09, 0·1 (Fig. 4*b*). The following Figs 5 and 6 illustrate the impulse characteristics which correspond to PRBS at $N = 63$ and $\Delta t = 8$ s. The noise added to the model response was at first non-correlated with q = 0, 0·01, 0·02, 0·03, 0·04, 0·05 (Fig. 5*a*) and with q = 0, 0·06, 0·07, 0·08, 0·09, 0·1 (Fig. 5*b*), then autocorrelated with q = 0, 0·01, 0·02, 0·03, 0·04, 0·05 (Fig. 6*a*) and with q = 0, 0·06, 0·07, 0·08, 0·09, 0·1 (Fig. 6*b*). If we examine the impulse characteristics in Figs 3–6, we can see that the added noise has not caused in any case such a distortion of the impulse characteristic which would

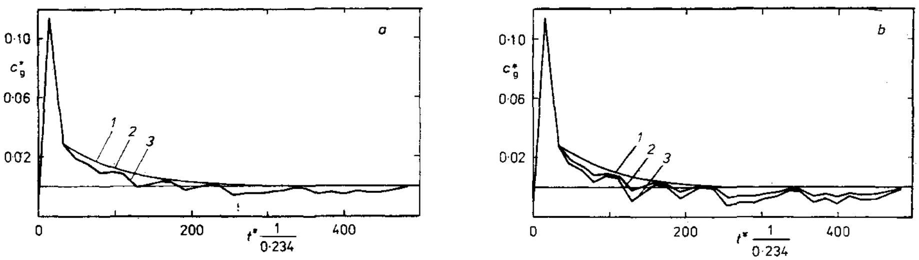

FIG. 3

The impulse characteristics found from the sum of response of model with mass transfer to the input signal of PRBS with $N = 31$, $\Delta t = 16$ s and non-correlated noise with different quotient q. *a* *1* $q = 0$, *2* $q = 0·01$, *3* $q = 0·05$. The impulse characteristics with $q = 0·02$, $q = 0·03$, and $q = 0·04$ are in the band delimited by the impulse characteristics with $q = 0·01$ and $q = 0·05$. *b* *1* $q = 0$, *2* $q = 0·06$, *3* $q = 0·1$. The impulse characteristics with $q = 0·07$, $q = 0·08$, and $q = 0·09$ are in the band delimited by the impulse characteristics with $q = 0·06$ and $q = 0·1$

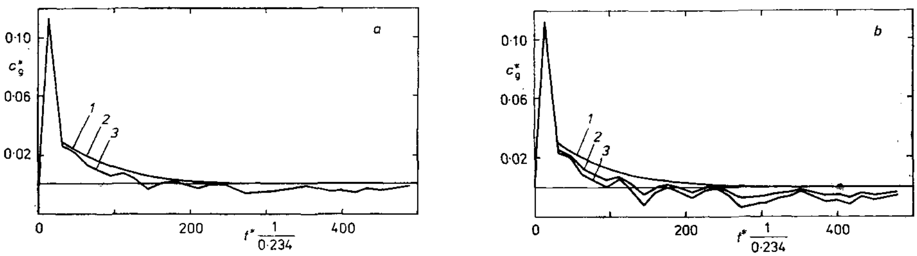

FIG. 4

The impulse characteristics found from the sum of response of model with mass transfer to the input signal of PRBS with $N = 31$, $\Delta t = 16$ s and autocorrelated noise with different quotient q. *a* *1* $q = 0$, *2* $q = 0·01$, *3* $q = 0·05$. The impulse characteristics with $q = 0·02$, $q = 0·03$, and $q = 0·04$ are in the band delimited by the impulse characteristics with $q = 0·01$ and $q = 0·05$. *b* *1* $q = 0$, *2* $q = 0·06$, *3* $q = 0·1$. The impulse characteristics with $q = 0·07$, $q = 0·08$, and $q = 0·09$ are in the band delimited by the impulse characteristics with $q = 0·06$ and $q = 0·1$

make the identification of the given model impossible. The effect of kind of noise has not been found out as well. A certain difference is, however, apparent if we compare Figs 3 and 4 with Figs 5 and 6. This difference in depicting the impulse characteristics is caused by their partial distortion owing to greater Δt. To exclude this distortion, we diminish Δt down to the integration step, i.e., 0·1 s. The corresponding value of N was 4 095. The calculated impulse characteristic for this PRBS agreed with the impulse characteristic calculated by direct way. Then we added, to the model response, the non-correlated noise with $q = 0$, 0·01 (Fig. 7*a*),

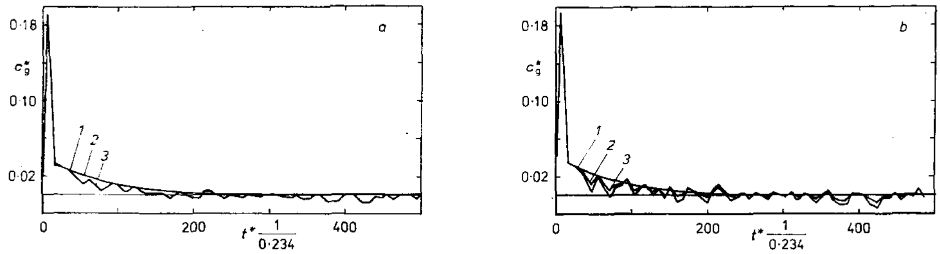

FIG. 5

The impulse characteristics found from the sum of response of model with mass transfer to the input signal of PRBS with $N = 63$, $\Delta t = 8$ s and non-correlated noise with different quotient q. *a* *1* $q = 0$, *2* $q = 0·01$, *3* $q = 0·05$. The impulse characteristics with $q = 0·02$, $q = 0·03$, and $q = 0·04$ are in the band delimited by the impulse characteristics with $q = 0·01$ and $q = 0·05$. *b* *1* $q = 0$, *2* $q = 0·06$, *3* $q = 0·1$. The impulse characteristics with $q = 0·07$, $q = 0·08$, and $q = 0·09$ are in the band delimited by the impulse characteristics with $q = 0·06$ and $q = 0·1$

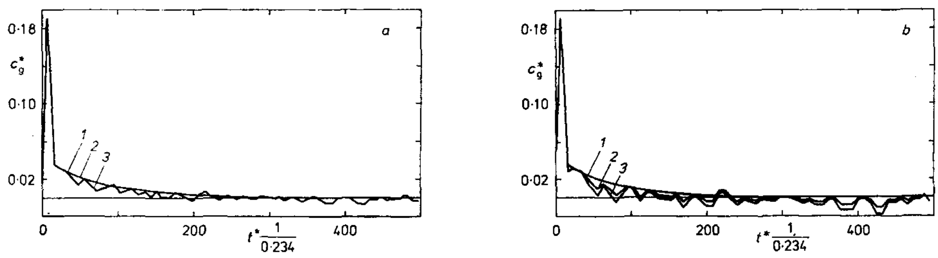

FIG. 6

The impulse characteristics found from the sum of response of model with mass transfer to the input signal of PRBS with $N = 63$, $\Delta t = 8$ s and autocorrelated noise with different quotient q. *a* *1* $q = 0$, *2* $q = 0·01$, *3* $q = 0·05$. The impulse characteristics with $q = 0·02$, $q = 0·03$, and $q = 0·04$ are in the band delimited by the impulse characteristics with $q = 0·01$ and $q = 0·05$. *b* *1* $q = 0$, *2* $q = 0·06$, *3* $q = 0·1$. The impulse characteristics with $q = 0·07$, $q = 0·08$, and $q = 0·09$ are in the band delimited by the impulse characteristics with $q = 0·06$ and $q = 0·1$

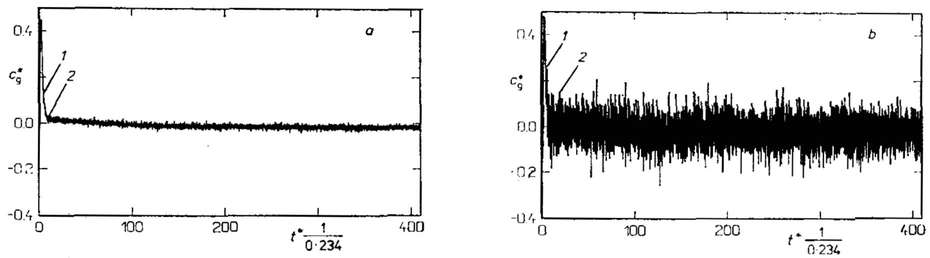

FIG. 7

The impulse characteristics found fiom the sum of response of model with mass transfer to the input signal of PRBS with $N = 4\,095$, $\Delta t = 0{\cdot}1$ s and non-correlated noise with different quotient q. *a* *1* $q = 0$, *2* $q = 0{\cdot}01$. *b* *1* $q = 0$, *2* $q = 0{\cdot}09$

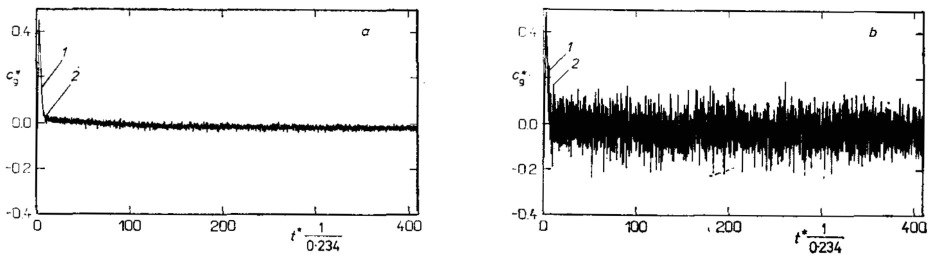

FIG. 8

The impulse characteristics found from the sum of response of model with mass transfer to the input signal of PRBS with $N = 4\,095$, $\Delta t = 0{\cdot}1$ s and autocorrelated noise with different quotient q. *a* *1* $q = 0$, *2* $q = 0{\cdot}01$. *b* *1* $q = 0$, *2* $q = 0{\cdot}09$

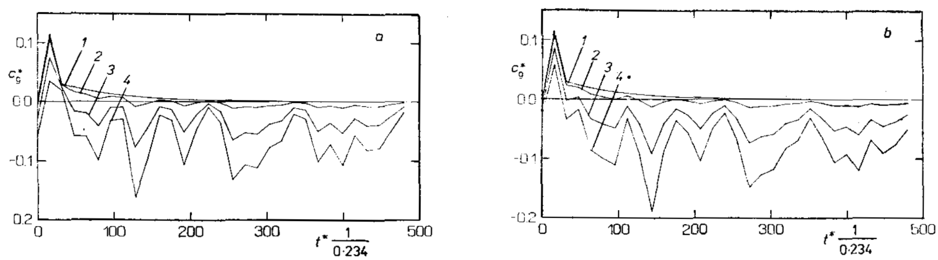

FIG. 9

The impulse characteristics found from the sum of response of model with mass transfer to the input signal of PRBS with $N = 31$, $\Delta t = 16$ s and non-correlated (*a*) or autocorrelated noise (*b*) with different quotient q. *1* $q = 0$, *2* $q = 0{\cdot}1$, *3* $q = C{\cdot}5$, *4* $q = 1$

$q = 0$, 0·09 (Fig. 7*b*) and the autocorrelated noise with $q = 0$, 0·01 (Fig. 8*a*), $q = 0$, 0·09 (Fig. 8*b*). The calculations for further values of q from the range 0 – 0·1 were not carried out. If we examine the impulse characteristics in Figs 7 and 8, it is apparent that even in the case of this PRBS, the presence of noise does not cause substantial distortion of impulse characteristic. Likewise the effect of kind of noise was not find out. By means of further calculations we investigated how the form of impulse characteristics would be influenced by increasing the quotient q up to the magnitude of driving force. At first we took the model with mass transfer and PRBS at $N = 31$ and $\Delta t = 16$ s. We chose three values of q (0·1, 0·5, 1) from the range of 0·1 – 1 and calculated the corresponding impulse characteristics. The impulse characteristics are drawn in Fig. 9*a*, calculation of which was carried out on adding non-correlated noise at $q = 0$, 0·1, 0·5, 1 at the output of the model. The autocorrelated noise of the same values of q was added at the output of model in case of impulse characteristics in Fig. 9*b*. For the model without mass transfer we chose PRBS at $N = 255$ and $\Delta t = 0{\cdot}075$ s. The value of Δt is sufficiently low so that the distortion of impulse characteristic owing to higher Δt does not take place. We added the non-correlated noise at $q = 0$, 0·1, 0·5, 1 to the model response without mass transfer and calculated the ordinates of impulse characteristics (Fig. 10*a*). Then we added the autocorrelated noise of the same values of q at the output of model and obtained the impulse characteristics in Fig. 10*b*. Both the figures 9 and 10 show that the adding of noise with the value of q greater than 0·1 results in more substantial distortion of impulse characteristic. On increasing quotient q, the impulse characteristic begins gradually to get lost in a band of pulsed values. This holds both for the model with mass transfer where greater Δt was used, and for the model without mass transfer where Δt was on the contrary very low. The effect of added noise was here the same regardless of the fact whether it was non-correlated or autocorrelated.

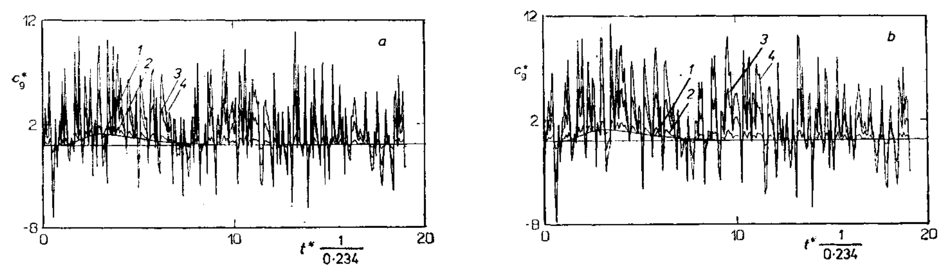

FIG. 10

The impulse characteristics found from the sum of response of model without mass transfer to the input signal of PRBS with $N = 255$, $\Delta t = 0{\cdot}075$ s and non-correlated (*a*) or autocorrelated noise (*b*) with different quotient q. *1* $q = 0$, *2* $q = 0{\cdot}1$, *3* $q = 0{\cdot}5$, *4* $q = 1$

CONCLUSION

In this work we investigated the simple probabilistic model derived from the deterministic model of bubble column by means of pseudo-random binary series of maximum length, PRBS. The simplicity of the probabilistic model consisted in the fact that all random phenomena influencing the system have been involved into the component of noise added to the output signal of model. The input signal of model was therefore formed by the ideal PRBS, the output signal was formed by the sum of the column response to this PRBS and non-correlated or autocorrelated noise.

In terms of the impulse characteristics calculated from these signals by the correlation method we investigated the probabilistic model on increasing amplitude of both kinds of noise. We delimited the value of quotient q at which the impulse characteristic was distorted to this extent that it was not possible to determine, according to its form, whether the model with mass transfer or without mass transfer was concerned. The effect of kind of noise on the form of impulse characteristics did not manifest itself in any case. This statement corresponds naturally only to the given comparatively small set of pseudo-random numbers representing the noise. Most simulation calculations were carried out for the axial dispersion model with mass transfer which was verified experimentally. With respect to the experience with PRBS used in experiments with bubble column[2], three PRBS with different values of N and Δt were chosen. For two PRBS, the impulse characteristics were partly distorted owing to greater Δt, however, it did not cause trouble when evaluating the impulse characteristics because the impulse characteristics with identical Δt were always compared in single figures. The simulation calculations of axial dispersion model without mass transfer were carried out for the only PRBS whose value of Δt did not cause distortion of impulse characteristic. If we summarize the given results, it follows from them that PRBS are suitable not only as experimental methods but can be applied also in simulating the behaviour of bubble column in presence of noise at its outlet.

SYMBOLS

$\bar{a}$	ordinate of PRBS
a_l	interfacial area per unit volume of liquid phase, $m^2\ m^{-3}$
c	concentration, mol m^{-3}
D	dispersion coefficient, $m^2\ s^{-1}$
$h(t)$	impulse characteristic
$k_{l(g)}$	mass transfer coefficient, m s^{-1}
L	height of bubble column, m
N	period of PRBS
$N(t)$	random signal (noise)
q	quotient of noise amplitude and distance of desired signal from centre to top

$R_{xy}(j\Delta t)$ pseudocorrelation function of PRBS and system response
t time, s
Δt basic time interval of PRBS, s
U real gas velocity, ms^{-1}
$x(t)$ input signal
$y(t)$ output signal
$\tilde{y}(t)$ output deterministic signal
z height coordinate, m
ε_g gas hold-up
χ linear equilibrium coefficient

Subscripts

g gas phase
l liquid phase
• dimensionless form
1 lower level of PRBS
2 upper level of PRBS

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